Stages in Multiplication

Multiplication - Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving **doubling**.

"If I have 3 dots on 1 side of my ladybird and double the number of dots, how many will it have?"

"Double 3 is 6."



Children will count two groups of the same size in practical contexts.

For example:





'Three apples for you and three apples for me. How many apples altogether?'

Using **real objects**, children will Count how many objects they have in each group and how many they have **altogether**.



Multiplication - Stage One

- Solve **one-step problems** involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Count in multiples of twos, fives and tens (to the 10th multiple)

Children will count **repeated groups** of the same size in practical contexts **using real objects**.

Use resources such as a **counting stick** to reinforce **repeated addition**.



They will use the vocabulary associated with multiplication in practical contexts. They will solve practical problems that involve combining groups of 2, 5 or 10. e.g. socks, fingers and cubes.



'Six pairs of socks.' 'How many socks altogether? 2, 4, 6, 8, 10, 12'

"6 groups of 2" "6 lots of 2"



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'Three pots of ten crayons.'
'How many crayons altogether? 10, 20, 30'
"3 groups of 10"
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"3 lots of 10"
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Use arrays to support early multiplication



'Five groups of two faces. How many faces altogether? 2, 4, 6, 8, 10'

'Two groups of five faces. How many faces altogether? 5, 10'



multiplication, with appropriate resources, throughout Stage one.

Use objects to consolidate

"Make 3 four times." times."





"Make 4 two

Use bingo pens to record

For example: "Stamp 5 six times."



Multiplication - Stage Two

- Recall and use **multiplication facts** for the 2, 5 and 10 multiplication tables
- Recall and use the following multiplication facts:

20	E.O	100
ZXU	520	IUXU
2x1	5x1	10x1
2x2	5x2	10x2
2x3	5x3	10x3
2x4	5x4	10x4
2x5	5x5	10x5
2x6	5x6	10x6
2x7	5x7	10x7
2x8	5x8	10x8
2x9	5x9	10x9
2x10	5x10	10x10

- **Calculate mathematical statements** for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs
- **solve problems** involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts
- show that multiplication of two numbers can be done in **any order**

Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures and diagrams

Use the **x** symbol to record **"times"** and **"multiplied by"** NOT to represent the language "lots of" or "groups of"

Combining Groups (repeated addition):

10 + 10 + 10 = 50







Children **must be secure** using multiplication in **practical contexts <u>before</u> moving onto using notation.**

Children must be explicitly taught that multiplication in an efficient way of showing repeated addition

To avoid confusion, especially with EAL children, be clear to model that 10×2 represents 10 multiplied twice, 6×10 represents six ten times. This is especially useful when rote learning timetables. Throughout this policy we have used this format to show the correct notation

Once children are secure with this representation, they should be taught to understand multiplication's commutative quality. During stage two for example they should be able to understand that 6×10 and 10×6 give the same answer enabling them to select the most efficient way for them to solve the problem.

Be sure that when introducing multiplication you make the distinction that "this is the number, this is what we are doing to it"

For example

10 x 2

"This is the number" (10) "This is what we are doing to it" (multiplying it by 2)

Using **arrays** to support multiplication:

$6 \times 5 = 30$ or $5 \times 6 = 30$

 $6 \times 5 = 30$ 6 + 6 + 6 + 6 = 30 $5 \times 6 = 30$ 5 + 5 + 5 + 5 + 5 = 30

To support children with arrays use concrete objects, pictoral representations (these could be linked to their particular interests):



Once familiar with real object arrays, show and encourage them to record: For example they could use the following:

Stamps

Stickers/dots





Continue to make the link to repeated addition



Multiplication - Stage Three

- Recall and use multiplication facts for the 3, 4 and
 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)
- Recall and use the following multiplication facts:

3x0	4x0	8x0
3x1	4x1	8x1
3x2	4x2	8x2
3x3	4x3	8x3
3x4	4x4	8x4
3x5	4x5	8x5
3x6	4x6	8x6
3x7	4x7	8x7
3x8	4x8	8x8
3x9	4x9	8x9
3x10	4x10	8x10
3x11	4x11	8x11
3x12	4x12	8x12

• Write and calculate mathematical statements for

multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, **using mental** and progressing to a formal written method

To support children with multiplication, the following resources should be used:

Cuisnaire rods Dienes





multiplication squares



Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to use **number arrays** to support multiplication, as appropriate (see Stage two quidance).

Partitioning method for multiplication of a two digit number by a one-digit number:

First, show this using the Dienes or cuisinaire



For example: "I have 16, 3 times"



30 + 18 = 48

 $10 \times 5 = 50$ $3 \times 5 = 15$ 50 + 15 = 65

To support children's understanding of the **partitioning method**, teacher could <u>demonstrate</u> by using an empty number line

However children **are not** expected to record in this way unless it supports their progression.

Again, use a counting stick alongside this method.



 $13 \times 5 = 65$



 $13 \times 8 = 104$

 $13 \times 8 = 104$

 $13 \times 8 = 104$

X	10	3
8	80	24

80 + 24 = 104(See addition policy for guidance on calculation methods)

'Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products (80 and 24) together.'

This will lead into expanded short multiplication:

Refine the recording in **preparation for formal short multiplication**:

Ensure children understand the true values they are multiplying

Formal short multiplication:	
13	Ensure that the digit 'carried over' is written under the line in the
<u>X 8</u> <u>104</u>	Use the language of place value to ensure
2 ← Always add the	understanding.
carried over value	

Continue to develop the formal written method of multiplication throughout stage three using two digit numbers multiplied by a one-digit number.

If children are confident progress to multiplying other two-digit numbers by a onedigit number (see Stage four guidance).

If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication Stage Four

- Recall multiplication facts for multiplication tables up to 12
 × 12
- Multiply two-digit and three-digit numbers by a onedigit number using **formal written layout**

Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Encourage children to use **mental methods** when dealing with greater values first.

Further develop the grid method for two-digit numbers multiplied by a one- digit number.

 $36 \times 4 = 144$

X	30	6
4	120	24

120 + 24 = 144 (add the partial products)See addition policy for guidance on calculation methods

digit number):

 $36 \times 4 = 144$



Refine the recording in preparation for **short**, **formal multiplication**

 $36 \times 4 = 144$

$$\begin{array}{r}
 3 \ 6 \\
 \underline{x \ 4} \\
 + 2 \ 4 \\
 \underline{120} \\
 14 \ 4
 \end{array}
 \begin{array}{r}
 (6x4) \\
 (30x4)
 \end{array}$$

This leads to **short multiplication (formal method)** of a two-digit number multiplied by a onedigit number:

36 x 4 =144	Use the language of place value to ensure understanding.
$3 6$ $X 4$ $\overline{144}$ 2	Ensure that the digit 'carried over' is written under the line in the correct column and then added not multiplied

Continue to practise the formal method of short multiplication of a two-digit number by a one -digit number throughout Y4.

If children are confident, continue to develop short multiplication with three- digit numbers multiplied by a one-digit number.

If necessary, return to the grid method and/or expanded method first:

 $127 \times 6 = 762$

x	100	20	7
6	600	120	42

600 + 120 + 42 = 762 (add the partial products) See addition policy for guidance on methods of calculation

This leads to expanded long multiplication

 $127 \times 6 = 762$

This will lead into **short multiplication (formal method)**:

127 <u>x</u> 6	Use the language of place value to ensure understanding.
<u>762</u> 14	Ensure that the digits 'carried over' are written under the line in the correct column
	and then added, not multiplied

If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication - Stage Five

 Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Build on the work covered in stage four with the formal method of short multiplication (two-digit number multiplied by a one-digit number).

When children are confident introduce multiplication by a two-digit number. If necessary, return to the grid method and/or expanded method first.

Grid method (two-digit number multiplied by a two digit number):

 $23x13 = (20+3) \times (10+3) = 299$

Encourage children to use mental strategies when dealing with greater values first For example 260 > 290 > 299

X	20	3	
10	200	30	230
3	3 60		69
L	260	1	Į

39

Add the partial products (200 + 30) + (60 + 9) = 299 OR 200+60+30+9 = 299(200 + 60) + (30 + 9) = 299 260 290 = 299

See addition policy for guidance on methods of calculation

Expanded long multiplication (two-digit numbers multiplied by a teen- number):

 $23 \times 13 = 299$

ALWAYS MOVING CHILDREN TOWARDS THE MOST EFFICENT METHOD

This leads into...

Compact long multiplication (formal method):

23x13 = 299

2 3 <u>X 1 3</u> + 6 9 (23x3) <u>2 3 0 (</u>23x10) 2 9 9

Use the language of place value to ensure understanding. Add the partial products.

Extend to larger two-digit numbers:

 $56 \times 27 = (50 + 6) \times (20 + 7) = 1512$

X	50	6	
20	1000	120	1120
7	350	42	392
	1350	162	1512

Add the partial products (1000 + 120) + (350 + 42) = 1512 or (1000 + 350) + (120 + 42) = 1,512

See addition policy for guidance on calculation methods

 $\overleftarrow{}$ Encourage mental strategies where efficient, dealing with greater values first For example

 $1000 \qquad 1350 \rightarrow 1450 \rightarrow 1470 \rightarrow 1512$

Expanded long multiplication (two-digit numbers multiplied by two-digit numbers):

I his expanded method is linked to the grid method.

This leads into.....

Compact long multiplication (formal method):

$56 \times 27 = 1512$



When children are confident with long multiplication extend with three-digit numbers multiplied by a two-digit number, returning to the grid method first, if necessary:

 $124 \times 26 = 3224$



Use the language of place value to ensure understanding. Emphasis the placing of **carried over values**

Add the partial products.

The prompts (in brackets) can be omitted if children no longer need them.

Extend with short and long multiplication of decimal numbers (initially in the context of money and measures), returning to an expanded method first, if necessary (see Stage six guidance).

If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication - Stage Six

• Multiply multi-digit numbers (including decimals) up to 4 digits by a two-digit whole numbers

Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to practise and develop the **formal short multiplication** method and **formal long multiplication** method with larger numbers and decimals throughout Stage six. Return to an expanded forms of calculation initially, if necessary (see stage five quidance).

The grid method (decimal number multiplied by a two-digit number):

 $53 \cdot 2 \times 24 = 1276 \cdot 8$

x	50	3	0∙2	
20	1000	60	4	1064· 0
4	200	12	0.8	212.8
	1200	72	4.8	1276· 8

The formal written method of long multiplication:

	5 3·2		
	x 24·0		It is an option to include $\cdot 0$ in this example, but
	2 1 ¹ 2 ·8	(53·2 x 4)	not essential.
	<u> 1064·0</u>	(53·2 x 20)	
_	1276.8		The prompts (in brackets) can be omitted if
			children no longer need them.

If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Stage six children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.