

# Stages in Subtraction

## Subtraction - Early Stages (EYFS)

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.

$$6 - 2 = 4$$



**'Take two apples away. How many are left?'**

Children will begin to count back from a given number.

## Subtraction – Stage One

- Given a number, identify **one less**
- Read, write and interpret **mathematical statements** involving subtraction (-) and the equals (=) sign
- **Subtract one- digit and two-digit numbers** within 20, including zero
- Solve **missing number problems** eg  $20 - ? = 15$

Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Children will continue to practise counting back from a given number.

Initially use a number track to count back for subtraction:



$$9 - 5 = 4$$

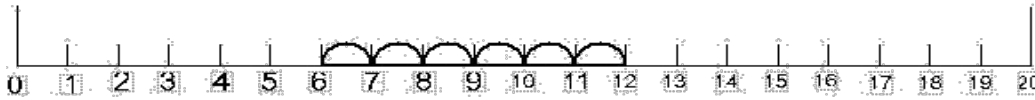
**'Put your finger on number nine. Count back five.'**

Use **objects** and **practical contexts** at this stage



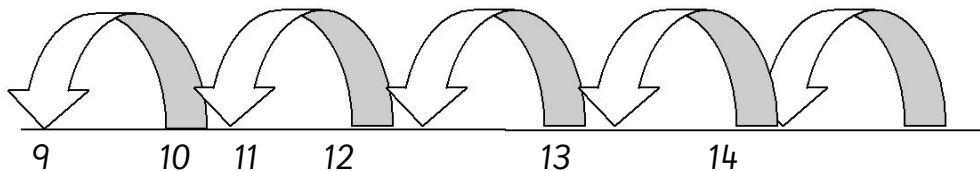
Once children are secure using the number track, progress to using a marked number line:

$$12 - 6 = 6$$



*'Put your finger on number twelve and count back six.'*

$$14 - 5 = 9$$



*'Put your finger on number 14 and count back five.'*

Ensure children are confident with using a **marked number line** before moving on to an **empty number line** (see stage two guidance).

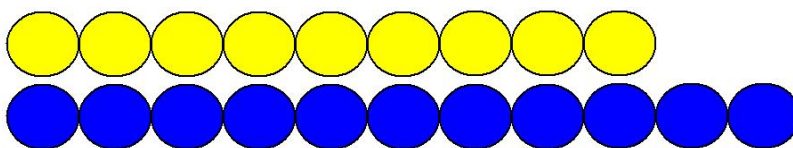
Continue to practise counting back for subtraction with numbers within 20.

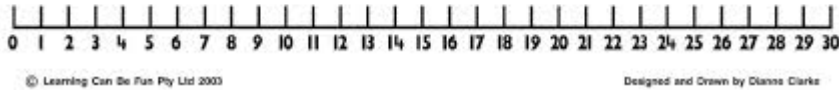
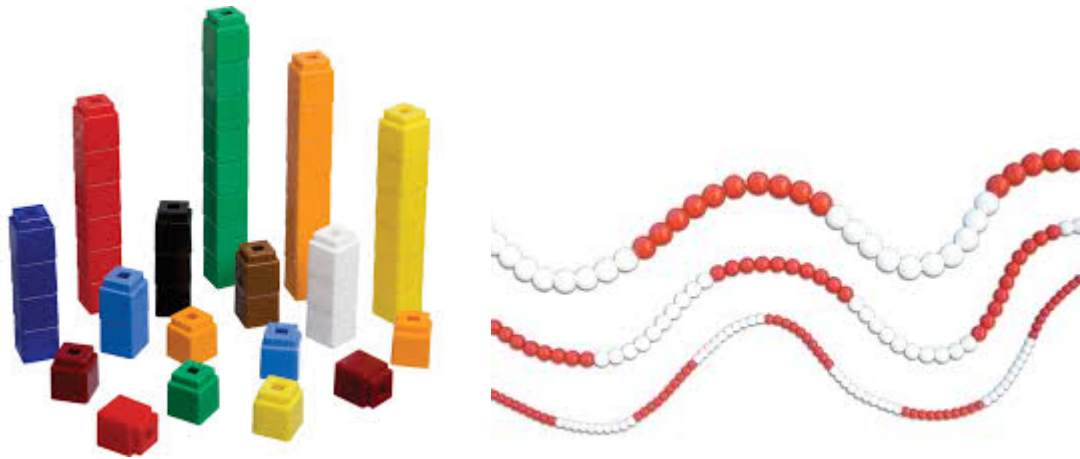
**Counting on to find a small difference:**

Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important here to understand the idea of "difference".

Count up from the **smallest number** to the largest to find the difference using resources, e.g. cubes, beads, number tracks/lines:

$$11 - 9 = 2$$





The *difference between* nine and eleven is two.

If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction – Stage Two

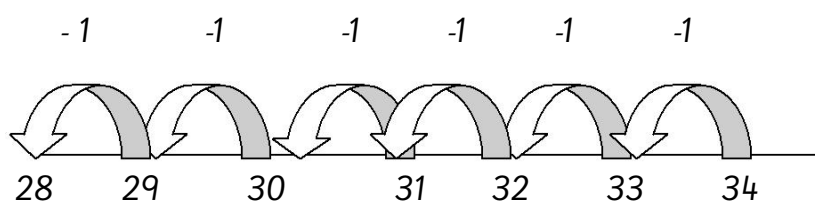
- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - o A two digit number and ones
  - o A two digit number and tens
  - o Two two-digit numbers

*Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.*

**Counting back** using an empty number line within 100,

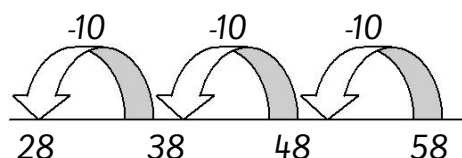
*in ones...*

$$34 - 6 = 28$$



...and in tens:

$$58 - 30 = 28$$



*Use in conjunction with a 100 square to show jumps of tens.*

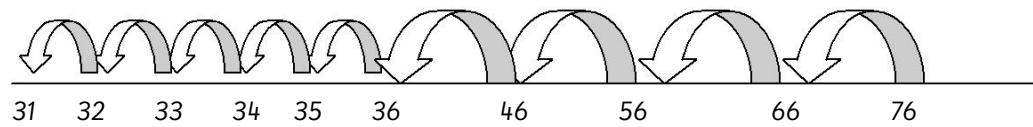
--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

*Subtraction, using partitioning, on an empty number line:*

$$76 - 45 = 31$$

$-1 \ -1 \ -1 \ -1 \ -1 \ -10 \ -10$

-10    -10



Use in conjunction with a 100 square to show jumps of tens and ones.

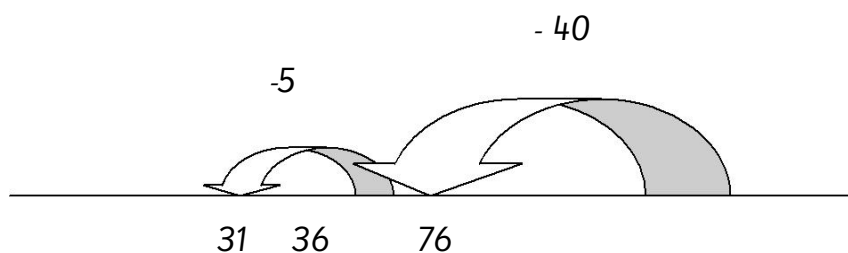
$$76 - 40 = 36$$

$$36 - 5 = 31$$

*Be sure to model subtracting the larger amount (the tens) before subtracting the smaller amount (the ones)*

If children are confident, they will begin to use more efficient jumps:

$$76 - 45 = 31$$



$$76 - 40 - 5 = 31$$

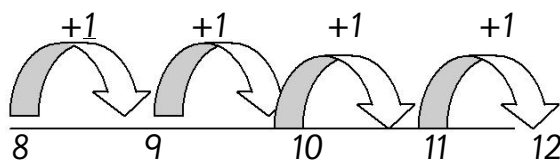
Again use in conjunction with a 100 square to show jumps of tens and ones.

### Counting on to find a small difference

Introduce complementary addition to find differences (only use for small differences). The use of **modelling using objects** is extremely important here to understand the idea of “difference” (see Stage one guidance).

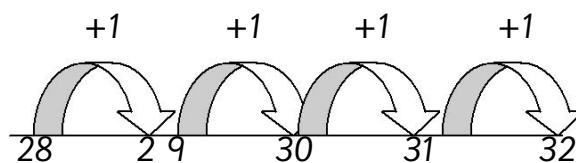
Count up from the **smallest number** to the largest to find the difference.

$$12 - 8 = 4$$



**‘The difference between 8 and 12 is 4.’**

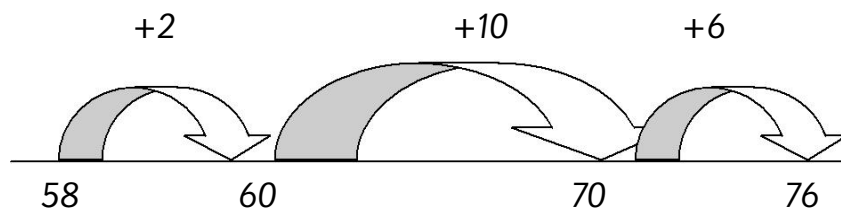
$$32 - 28 = 4$$



**‘The difference between 28 and 32 is 4.’**

If children are confident, further develop this method:

$$76 - 58 = 18$$



*'The difference between 58 and 76 is 18.'*

Ensure children are confident and secure with this method before moving onto subtracting with numbers that bridge 100.

For numbers bridging 100 use a 200 grid to support.

1 TO 200									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

If, at any time, children are making significant errors, **return to the previous stage in calculation.**



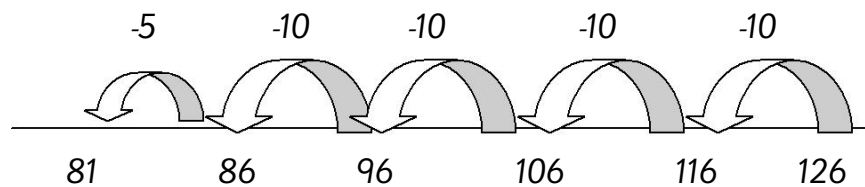
### Subtraction – Stage Three

- Subtract numbers with up to three digits, using **formal written method** of column subtraction

Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

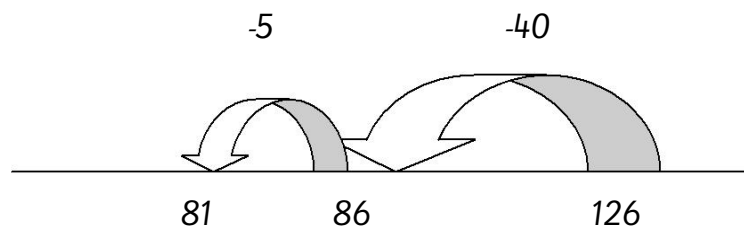
Further develop the use of the **empty number line** with calculations that bridge 100:

$$126 - 45 = 81$$



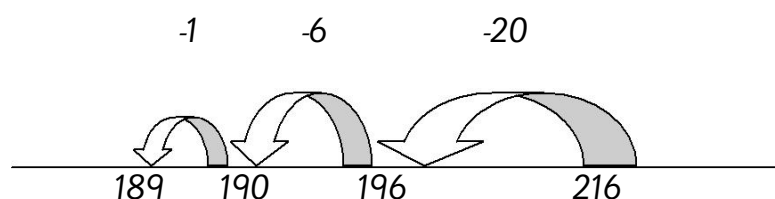
Use a 200 grid to support counting back in tens and bridging 100

Then use more efficient jumps:



Extend with larger numbers by **counting back**...

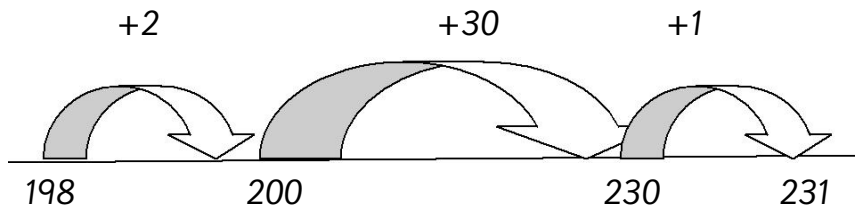
$$216 - 27 = 189$$



...and by **counting on** to find the difference (small

difference):

$$231 - 198 = 33$$



**'The difference between 198 and 231 is 33.'**

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns). Use two-digit numbers when introducing this method, initially:

$$78 - 23 = 55$$

$$\begin{array}{r} 70 + 8 \\ - 20 + 3 \\ \hline 50 + 5 = 55 \end{array}$$

**'Partition numbers into tens and ones/units. Subtract the ones, and then subtract the tens. Recombine to give the answer.'**

In this example decomposition (exchange) is not required.

You might replace the **+** sign with the word **'and'** to avoid confusion.

This will lead into the **formal written method**:

$$\begin{array}{r} 78 \\ - 23 \\ \hline 55 \end{array}$$

Use the language of **place value** to ensure understanding:

**'Eight subtract three, seventy subtract twenty.'**

A **number line** would be an appropriate method for this calculation but use two-digit numbers to **illustrate the formal written method initially**.



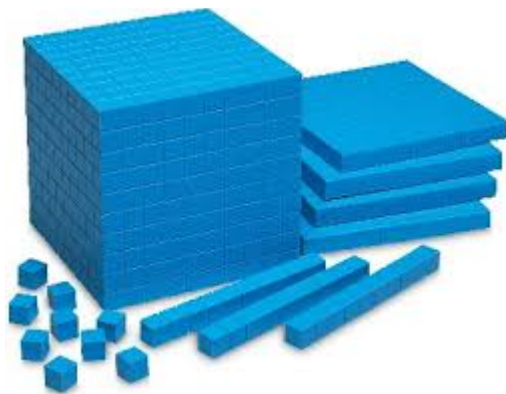
$$73 - 27 = 46$$

$$\begin{array}{r} 70 + 3 \\ - 20 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 60 + 13 \\ - 20 + 7 \\ \hline 40 + 6 = 46 \end{array}$$

73 is partitioned into 60+13 in order to calculate 73-27

Children will need to **practise partitioning numbers** in this way. Base- ten materials should be used to support this so the children have a concrete understanding.



When children are confident with the expanded method introduce the **formal written method**, involving **decomposition/exchange**:

$$\begin{array}{r} 73 - 27 = \\ 46 \end{array}$$

$$\begin{array}{r} 6 \ 13 \\ 73 \\ - 27 \\ \hline 46 \end{array}$$

Use the language of **place value** to ensure understanding.

**'We can't subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.'**

Again use base ten materials to support understanding.

If children are confident, **extend the use of the formal written method** with numbers over 100, returning to the expanded method first, if necessary.

Only move onto this method once children are secure using numbers under 100.

$$235 - 127 = 108$$

$$\begin{array}{r} 2\ 15 \\ 2\ 3\ 5 \\ -1\ 2\ 7 \\ \hline 1\ 0\ 8 \end{array}$$

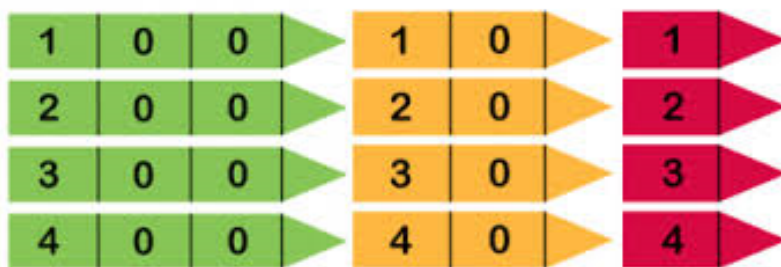
Use the language of place value to ensure understanding.

In this example it has only been necessary to exchange from the tens column.

Use base ten materials to support understanding.

If, at any time, children are making significant errors, return to the previous stage in calculation.

Use place value cards to support children at this stage



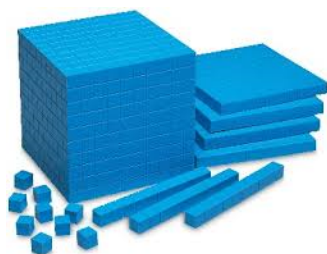
## Subtraction – Stage Four

- Subtract numbers with up to 4 digits using the **formal written method of columnar subtraction** where appropriate

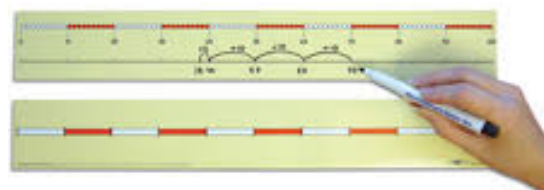
Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to teach the use of empty number lines with three and four digit numbers, as appropriate.

Continue to develop the formal written method of subtraction by revisiting the **expanded method first**, if necessary. Continue to use base-ten materials to support understanding.



1	0	0	1	0	1
2	0	0	2	0	2
3	0	0	3	0	3
4	0	0	4	0	4



$$258 - 73 = 185$$

$$\begin{array}{r} 200 + 50 + 8 \\ - \quad 70 + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 100 + 150 + 8 \\ \text{becomes } - \quad 70 + 3 \\ \hline \end{array}$$

$$100 + 80 + 5 = 185$$

You might replace the **+** sign with the word '**and**' to avoid

confusion. Children will need to practise partitioning in a variety of ways.

This leads to the **formal written method**, involving decomposition...

$$\begin{array}{r} 1 \text{ } 15 \\ 258 - \\ \underline{73} \\ 185 \end{array}$$

Use the language of place value to ensure understanding.

In this example it has been necessary to exchange from the hundreds column.

Ensure children are **exchanging** and using this language when being taught this method. When exchanging, ensure they write the new value at the top of their calculation **as in the above example**.

Further develop by subtracting a three-digit number from a three-digit number:

$$637 - 252 = 385$$

Ensure that children are confident in partitioning numbers in this way.

$$\begin{array}{r} 600 + 30 + 7 \\ - 200 + 50 + 2 \end{array}$$

$$\begin{array}{r} 500 + 130 + 7 \\ - 200 + 50 + 2 \\ \hline 300 + 80 + 5 = 385 \end{array}$$

This leads to a **formal written method**:

$$\begin{array}{r} \phantom{5}13 \\ 6\cancel{3}7 \\ - \underline{252} \\ 385 \end{array}$$

Use the language of place value to ensure understanding and use base-ten materials to support

When children are confident, develop with **four digit numbers** and decimal numbers (in the context of money and measures).

$$3625 - 1219 = 2406$$

$$\begin{array}{r} \phantom{1}15 \\ 36\cancel{2}5 \\ - \underline{1219} \\ 2406 \end{array}$$

If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction - Stage Five

- Subtract whole numbers with more than 4 digits, including using **formal written method** (column subtraction)

Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to teach the use of empty number lines with larger numbers and decimals, as appropriate.

Continue to **develop the formal written method for subtraction** with three and four digit numbers (see Stage four guidance), **returning to an expanded method** and using base ten materials, if necessary.

$$503 - 278 = 225$$

$$\begin{array}{r} 500 + 0 + 3 \\ - 200 + 70 + 8 \\ \hline \end{array} \quad \begin{array}{r} 400 + 90 + 13 \\ - 200 + 70 + 8 \\ \hline 200 + 20 + 5 \end{array}$$

In this example 503 has to be partitioned into  $400 + 90 + 13$  in order to carry out the subtraction calculation.

This leads into the formal written method (there is potential for error in this example):

$$\begin{array}{r} \phantom{4} \phantom{9} \phantom{13} \\ \cancel{5} \cancel{0} \cancel{3} \\ - 278 \\ \hline 225 \end{array}$$

There are no tens in the first number (503) so we have to exchange a hundred for 10 tens before we can exchange a ten for ten ones/units

It would be appropriate to **discuss the use of mental calculation methods** with an example like this one, i.e. would an empty number line be a more efficient method for these numbers?



When children are confident **extend with larger numbers** (and **decimal numbers**). Return to an expanded method, if necessary.

$$12731 - 1367 = 11364$$

$$\begin{array}{r} \phantom{1}^6 \phantom{2}^{12} \phantom{3}^{11} \\ 12\cancel{7}3\cancel{1} \\ - \phantom{1}1367 \\ \hline 11364 \end{array}$$

In this example it has been necessary to exchange from the tens and the hundreds columns.

If children are making significant errors, provide calculations where only one exchange is required.

Introduce **subtraction of decimals**, initially in the context of **money and measures**.

$$£166.25 - £83.72 = £82.53$$

$$\begin{array}{r} 16 \quad 5 \quad 12 \\ \cancel{166}.\cancel{2}5 \\ - 83.72 \\ \hline 82.53 \end{array}$$

**Ensure the decimal points line up.**

Continue to practise and apply the formal written method with large numbers and decimals throughout stage five.

If, at any time, children are making significant errors, return to the previous stage in calculation.

### Year Six - Subtraction

No objectives have been included in the programmes of study explicitly related to written methods for subtraction in stage six. However, there is an expectation that children will continue to practice and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous stages guidance for methods).

Our aim is that by the end of stage six children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.